Linear Programming in the Community

In this project, we aimed to help a food pantry service by selecting the optimal combinations of food items in order to feed the most people for the longest amount of time. We set down criterions of how much nutrition an average audience of the pantry service needs to be counted as being well fed for a day, and we attempted to maximize this number of days, subjected to the constraint of the budget of the pantry, the recommended healthy range of daily nutritional categories that each person should take and the amount of discounted food available for purchase.

Backgrounds

Our target community organization is the Wesley Evening Food Pantry. This organization provides free food to impoverished families, on every third Thursday of each month. Many families may have trouble buying food near the end of each month, before the next paycheck. This food pantry fills in for this shortfall. Unlike other food pantry services, where they distribute food by packets, each containing about the same selection of items, the Wesley food pantry purchases their food items all together first, and then distributes the food based on the personal preferences of the target members of the community, through a lottery system.

Unlike other community food services where they distribute only the food donated to them or they have readily available on hand, the Wesley Evening Food Pantry purchases their food through Eastern Illinois Food Bank (EIF), thus allowing some degree of flexibility to choose the best selection of food. EIF is a non-profit organization collecting donated or discounted food, from various sources, such as farmers having surplus produces that they can not sell, supermarkets donating their surplus stocks, food drives organized by churches, etc. Food pantries, soup kitchens, homeless shelters from all over Eastern Illinois orders discounted food from EIF, with both parties (the EIF and the distributor organization) sharing the cost of the food. EIF has a daily shopping list, which represent their current inventory of items with price listed as shared maintenance. The shopping list has minor changes daily (new item arrived or popular item sold out), and the price of each item may fluctuate up and down, based on supply and demand. The monthly food purchase does not happen all on one day. The director of the Wesley food pantry, Ms. Donna Camp, monitors this list daily to look for good deals to purchase. The food items distributed at each month's pantry are not the same, which depends heavily on which items are available at EIF.

Linear Program Setup

Our research topic is to find out the best grocery selection so that the Pantry can present the best selection of food items to the community. The food pantry would like to provide the widest selection of healthiest, most nutritious food to the most people, subject to budget constraints of a non-profit organization. This project is therefore concerned with optimizing the best selection of food items (what items should be chosen and in what quantity) to best service the community, while balancing the spectrum of nutrition delivered and allocating excesses for the healthiest nutritional categories, such as protein.

Since the available selection list is subjected to change, we chose the food orders list from last month's pantry to get the exact pool of available items to select from. We took a snapshot of the EIF shopping list on May 7th [1] to get the price of the items, since the shopping list changes daily and we want one fixed price for the entire analysis. For the items that does not exist anymore on the shopping list, we went to the websites of supermarkets, such as Walgreen, Wal-Mart, etc., to take down the market price, and as a heuristic, divided it by 3, to get the estimated discounted price at EIF (this heuristic is developed from observing the things on EIF list usually cost 1/3 the price from supermarkets). This study will find out how much last month's purchase decisions could be improved by using linear programming techniques, delivering more nutrients to the community, while using the same budget. Through this, we can find out the sensible constraints to use to ensure a diverse range of food selection and to discover which nutrients are usually the hardest to come by. Changing the linear program setup to plan for next month's purchase merely involves updating the prices and units available with otherwise the exact same set of constraints.

Variables and Constraint Equations

Each variable in our linear program represents the quantity of each item that is available for distribution at the monthly pantry day. From the food order list last month, there are 43 food items. Each item is available from EIF in packs. For example, Tuna is purchased by 48 cans at a time. We treat each pack as a single unit for each variable. Thus we are solving for an Integral linear program.

The total price of the food items is constrained by the budget, which is our first constraint equation. Every month, Wesley Food Pantry uses about $3,000-5,000 to purchase the food. According to the director of the pantry, Ms. Camp, there is no fixed monthly budget, and instead in last year as a whole, the pantry used $52,000 [2], which comes out to be about $4,333 per month. Using the food order list from last month and the price that we collected, we calculated that total price of all the food available at last month's pantry is $9290.34. Such a large price probably implies there was a lot of leftover food from two months before. Thus in our study, we bounded our budget above by $9290.34. Since this is a comparative study, it does not matter that this budget is unlikely to be available this month. We merely want to see whether our linear program can pick another selection of food that will increase the nutritional values by a large amount, with the exact same price, items and availability constraints, so to demonstrate the merits of our method.

We selected 11 categories of nutrition that exist on every nutrition fact labels: Total Calories, Total Fat, Saturated Fat, Cholesterol, Sodium, Total Carbohydrates, Protein, Vitamin A, Vitamin C, Calcium, and Iron. For each food item, we find out each of the 11 nutritional facts (mostly from source [3], but if we cannot find it on that site, we went to the local county market to find an product as close to the item as possible and take down its facts). Each of these 11 categories forms some constraint equations, either requiring a minimum or maximum provision of nutrients (more explanations later).

We also added another variable called Days, which represent the number of days the sum total nutrition in each category can be last for an average pantry client, who use up daily recommended nutrition every day. As an example the form of constraint equation for calories is

$$(Calories\ of\ item\ 1)x_1 + (Cal.\ of\ item\ 2)x_2 + \cdots + (Cal. of\ item\ 43)x_{43} \\ - (Leeway)(Daily\ recommended\ Cal. Consumption)D_{ays} \geq 0$$

which required that the total calories to be distributed at the pantry to be above a minimum of calorie requirement. (The Leeway constants will be explained later.) The cost function is only one variable. We maximize for the Days variable.

We researched what an average adult and children should ideally intake in a day. According to, Ms. Camp, the composition of the monthly clients of the pantry is about 60% adults and 40% children, with the gender distributed evenly, based on statistics from 2010. We assumed that this fact have not changed dramatically in 2011. This demographics distribution has a heavier proportion of children than the general population, since the general population has about 25% children [4]. (This corresponds well to the fact that families having many children have a harder time to provide food.) Thus we decided not to directly take the FDA recommended daily values, and instead do a weighted average for the daily nutrition requirements for the average audience of the Wesley food pantry.

For each of the 11 nutrition categories, an average pantry client needs a daily value of 0.6 * (adult daily value, given by FDA, [5]) + 0.4 * (child daily value, given by Dietary Reference Intakes (DRI) from the Institutes of Medicine, [6])

The second source provided the recommended daily value for children in each age range. We assumed that people below 18 are categorized as children and the age of the children visiting the pantry are even distributed (e.g. there are equal amount of 2 year olds as there are 15 year olds). The average nutrition constrains for children is calculated as following: (NV=Nutrition Value, NV(x, y) represents recommended daily NV for children between x and y age.)

$$\mathrm{NV} = \frac{\mathrm{NV}(2,3) * 2 + \mathrm{NV}(4,8) * 5 + \mathrm{NV}(9,13) * 5 + \frac{[\mathrm{NV}(14,18,\mathrm{girls}) + \mathrm{NV}(14,18,\mathrm{boys})]}{2} * 5}{17}$$

The result of the calculation is given in the table below:

Recommend Daily Values	Adult	Children	Weighted Average
Calories	2000	1773.53	1909.41
Total fat (g)	65	62.91	64.16
Saturated fat (g)	20	19.88	19.95
Cholesterol (mg)	300	300	300
Sodium (mg)	2400	1705.88	2122.35
Carbohydrate (g)	300	220	268
Protein (g)	50	30.65	42.26
VA (IU)	5000	18.77	3007.51
VC (mg)	60	42.94	53.18
Calcium (mg)	1000	1058.82	1023.53
Iron (mg)	18	10.53	15.01

The weighted averages are used as the (Daily Recommended … Consumption) coefficients for the Days variable in the constraint equations.

The pantry lets each person to select their food item, giving each person a random number for each item they want. Each person is allowed to take turns to make their pick of the items they want, based on the order of their numbers. Thus some food items maybe more popular than others and ran out very quickly, and our planned nutritional values may not get distributed evenly, because people pick their food more based on taste and personal preference than a strict desire to maximize and balance their nutritional intakes. We only hope to provide a maximal overall nutritional value to the community, and we planned for the cases where people picked food that cause a nutritional imbalance, such as picking too much fatty food out of the selections. Thus we prepared excesses (above the recommended daily values) in beneficial nutrition categories, such as Vitamin A, Vitamin C, proteins, etc., and lowered the supplies of nutrition categories where people in the modern society can easily get a harmful excess of, such as saturated fat, cholesterol, sodium, etc.

≤	Lower and Upper Limits	≤
0.4	Weight of food (lb)	1.6
0.8	Calories	1.5
0.5	Total fat (g)	1
0	Saturated fat (g)	0.8
0	Cholesterol (mg)	0.5
0	Sodium (mg)	1
0.8	Carbohydrates (g)	1.5
1.5	Protein (g)	2.5
1	Vitamin A (IU)	2.5
1	Vitamin C (mg)	2.5
1	Calcium (mg)	2.5
0.75	Iron (mg)	1.5

In our constraint equations, we set upper limits and lower limits for each nutrient, using the (Leeway) constants. The upper and lower limits specify the percentages of the total provided nutrients are allowed to exceed or go below the recommended values.

Suppose we have picked enough food to fill a person's calories need for 10 days, which is about 20,000 Calories. Using our daily values above, there should ideally be about 650 grams of total fat in the food that we selected. Our lower and upper limits on fat require that there should be minimally 325 grams and maximally 650 grams of fat in our selection. If the total fat value of our selection violates the limits, by either having too much or too little fat, then our selection is disallowed, because it provides an imbalance of nutrition.

These Leeway constants are picked by feel and do not have much justifications behind them. They are selected based on how much we perceived that they are harmful to the body, (such as cholesterol which is limited to at max 50% of daily value), or beneficial to the body, (such as protein which we requires to be 150% of daily value). They are also partly selected by trial and error, so that the LP would not be infeasible and the nutrition requirements would not be so strict that maximal of our Days variable would be too small.

Besides the nutrition constraints, we also have one other category to use for the constraint equations. One may have a meal that is full of nutrition and calories, but if the weight of the dinner is light, the person is not going to feel full. Based on research, we found that an average American consume about 5 pounds of food per day [7]. Thus we set 5 pounds as the daily recommended weight of meal. However, we left a huge Leeway constant, such that anywhere from 2 pounds to 8 pounds can be a legitimate weight of meal. This is because we expect people will not solely rely on the pantry as their food source, and they would have filler food, such as bread or soup at home to eat with the meal. However, this constraint is still useful, as it prevents purchase of vast quantity of soy nuts or other nutritionally concentrated but otherwise “cannot be expected as the main course” food.

Our last set of constraints is to guarantee diversity of food items. We required that the LP may not select more than 50 units in each kind of items. This matches well with the buying patterns of last month, as most of the food items have only double digits in quantity, thus ensuring similar level of food diversity. Without these constraints, the linear program would select hundreds of only a few food items that somehow match particularly well our criterions, but otherwise is an undesirable selection because the monotony of the food or the items all being condiments, etc…

Solution to the ILP

Optimizing the ILP with the constraints and cost function described above results in the solution vector as shown in the table below. Wesley Food Pantry's last month's purchase of food is also listed below for comparison.

Items	Last Month Purchase	ILP Optimized Purchase
Tuna / 48 Cans (Limit 1 tuna per person in household)	23	0
Pasta O's in Tomato or Cheese Sauce / 24 Cans	62	50
Mac N Beef in Tomato Sauce	50	50
Beef Ravioli / 24 Cans	23	0
Tuna Skillet Dinner	29	0
Cheeseburger Skillet Dinner / 12 Boxes	18	0
Beef Pasta Skillet Dinner	16	0
Chicken Noodle Soup / 24 Cans	26	0
Vegetable Soup w/Beef Stock	9	0
Macaroni & Cheese / 24 Boxes	2	50
Pinto Beans / 24 Cans	42	50
Chili Beans /24	25	44
Dried green peas	2	0
Dried white beans	14	50
Roasted Soy Nuts	7	50
Peanut Butter / 12 Jars	33	50
Strawberry Preserves / 12 Jars	15	0
Grape Jelly	8	0
Green Beans / 24 Cans	21	50
Sliced Carrots / 24 Cans	20	0
Mixed Vegetables / 24 Cans	20	50
Fruit Cocktail / 24 Cans	50	29
Peaches / 24 Cans	26	50
Corn &/or Asparagus &/or assorted veggies / 24 Cans	4	50
Spaghetti Sauce / 24 Cans	9	14
Elbow &/or Curly Macaroni / 20 Boxes	22	0
Pasta Spirals (Quick Cook)	25	0

Instant Rice / 4 Bags	68	11
Long Grain Rice (24 cans)	12	0
Cornbread Stuffing Mix	11	0
Triscuits / 12 Packages	15	0
Chewy Granola Bars / 12 Box / 10 Bars	15	50
Cereal-Bran Flakes / 12 Boxes	21	50
Cereal-Crispy Rice / 12 Boxes	26	0
Toasted Oats Cereal / 12 Boxes	40	49
Instant Oatmeal-Assorted / 12 Boxes	20	50
Assorted Baby Food from Sharing Shelves	15	5
Graduates Cheddar Corn Snack (Whole Grain) / 8 Pouches	60	0
Mustard / 8 bottles	3	50
Mayo / 12 bottles	4	0
Salad Dressing (Like Miracle Whip) / 12 Jars	20	0
Pork and Beans	42	50
Apples - Bags	150	49

According the metric we used to decide how much nutrition is enough to healthily sustain a person each day, $D_{ays} = 6987$, in the ILP solution. This means that if an average pantry client, who has 60% adult nutritional need and 40% child nutritional need, picked out the most balance selection for him/herself, then he/she can sustain for 6987 days just on the food from the pantry. Thus since there were 1,044 people who visited the last month, if we assume that each person got the same amount of food, then they can last about a week.

To calculate the corresponding Days variable based on how food was actually purchased last month, we plugged in the unit packages purchased for each item into the constraint matrix and we calculated the maximum value of the Days variable allowable by the constraint matrix. D_{ays} =3383, which means if food was distributed evenly among the pantry clients last month, the food will last for a little more than 3 days for everyone, assuming that they only consume the pantry food, and they want to eat healthily and fully every day.

Conclusion

Even though our requirements for a healthy daily nutritional consumption is higher than what is practical for most people, and our upper and lower limits of nutrition are somewhat arbitrary and made to suit our purpose, we have demonstrated that using linear programming, a healthier and more nutrition filled food selection can be served to the community. Using the same amount of money and selecting from the same pool of food items, we were able to double the nutrients delivered to the community, enabling families to last twice as long using the food received from the pantry. Further improvement for the project should be directed towards finding more sophisticated constraint equation that will guarantee a wider and diverse selection of food. (For example, the ILP optimized solution suggested 50 packages of mustard, each containing 8 bottles. No one is going to live on mustard just to live healthier.)

We suggest a hybrid approach to select food to deliver maximum amount of nutrition to the community, while ensuring that the food selection does not lack common sense. Throughout the month, one should first manually purchase cheap deals and popular items as they appear in the EIF shopping list. Then, when the pantry day nears, to meet the quota, one can constrain each of the final selection of items to be at least more than what has already been purchased, and the same ILP can be used to optimize how to distribute the rest of the money on food purchase.

Citations

1) “Agency Shopping List." *Eastern Illinois FoodBank*. Eastern Illinois FoodBank, 07 May 2011. Web. 7 May 2011. <http://eifoodbank.org/wp-content/uploads/2009/09/EIF-Shopping-List-2229.htm>.

2) Camp, Donna. "Re: math482." Message to Brian Wang. 04 May 2011. E-mail.

3) *SelfNutritionData*. CondéNet, 12 May 2011. Web. 12 May 2011. <http://nutritiondata.self.com/>.

4) “America's Children in Brief: Key National Indicators of Well-Being, 2010." *ChildStats.gov*. N.p., n.d. Web. 12 May 2011. <http://www.childstats.gov/americaschildren/tables.asp>.

5) “Appendix F: Calculate the Percent Daily Value for the Appropriate Nutrients." *US Food and Drug Administration*. US department of Health & Human Services, 09 May 2011. Web. 12 May 2011. <http://www.fda.gov/Food/GuidanceComplianceRegulatoryInformation/GuidanceDocuments/FoodLabelingNutrition/FoodLabelingGuide/ucm064928.htm>.

6) "Daily Values vs. Nutritional Recommendation for Children." *USDA/ARS Children's Nutrition Research Center at Baylor College of Medicine*. Baylor College of Medicine, 24 May 2004. Web. 12 May 2011. <http://www.bcm.edu/cnrc/consumer/archives/percentDV.htm>.

7) Pimentel, David, and Mario Giampietro. "HIGHLIGHTS OF "FOOD, LAND, POPULATION, AND THE U.S. ECONOMY"." N.p., 21 November 1994 . Web. 12 May 2011. <http://jayhanson.us/page40.htm>.

Explanation of the appendix

The constraint equations are presented in excel sheet Constraint_Equations.xlsx in the appendix. Every constraint equations are there, except the max of 50 packages per item on every item or that amount of packages needs to be positive.

The excel sheet where we collected all the nutrition information is attached in the appendix, in Wesley_Datasheet.xlsx. We first collected calories, total fat, etc. per serving. Then we find out how many servings there are for each unit package, and multiply the values by the number of servings per package. Thus, the values presented on the sheets are the amount of nutrition per package, not per servings. (This is why one sees a huge number of calories in Tuna, for example, because it is given as the total calories of 48 can of tuna at a time.)

The Mathematica notebook file that we provided contains the setup of the linear programmer solver. In that file, there is the cost vector, c, the constraint matrix, m, and the constraint values, b. These symbols are chosen to be similar to our class conventions. The LinearProgram function tries to minimize the cost function. The m matrix is imported straight from the highlighted region of Constraint_Equations.xlsx. The last column of that matrix is the Days variable. The r matrix is the solution vector. This vector is copied into the first two columns of Wesley_Datasheet.xlsx.